# Opening the Black Box: Structural Factor Models versus Structural VARs

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**Abstract**

In this paper we study identification in dynamic factor models and argue that factor models are better suited than VARs to provide a structural representation of the macroeconomy. Factor models distinguish measurement errors and other idiosyncratic disturbances from structural macroeconomic shocks. As a consequence, the number of structural shocks is no longer equal to the number of variables included in the information set. In practice, the number of structural shocks turns out to be small, so that only a few restrictions are needed to reach identification. Economic interpretation is then easier. On the other hand, with factor models we can handle much larger information sets—virtually all existing macroeconomic information. This solves the problems of superior information and fundamentalness and enables us to analyze the effects of the shocks on all macroeconomic variables. In the empirical illustration we study a set of 89 US macroeconomic time series, including the series analyzed in the seminal paper of King *et al.* (1991). We find that the system of impulse response functions of these series is non-fundamental and therefore cannot be estimated with a VAR. Moreover, unlike in King *et al.* (1991), the impulse response functions of the permanent shock are monotonic and therefore more credible if the permanent shock is interpreted as technical change.

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# 1 Introduction

Structural VARs and related models like the Structural ECM have become the basic analytical framework for a large part of modern Macroeconomics. Macroeconomic variables are represented as driven by serially uncorrelated shocks, each having a different source or nature, like "demand", "supply", "technology", "monetary policy" and so on. Each variable reacts to a particular shock with a specific sign, intensity and lag structure, summarized by the so called "impulse-response function". Such response functions can be recovered by imposing suitable identifying restrictions. Implications of economic theory not used for identification can then be compared with estimation results and tested.

In the recent literature, some important shortcomings of this successful research paradigm have been highlighted. A partial list includes Hansen and Sargent, 1991, Lippi and Reichlin, 1993, 1994, Faust 1998, Leeper, Sims and Zha 1996, Christiano, Eichenbaum and Evans, 1999, Cochrane, 1998, Rudebush, 1998, Sims, 1998, Uhlig, 1999. For a review see Stock and Watson, 2001. Major problems are: (i) the fundamentalness assumption, which is needed for identification, is essentially arbitrary, particularly for small VARs; (ii) results are very sensitive to the choice of the variables to include in the system; (iii) the identifying restriction are often arbitrary, particularly in large systems. Let us briefly illustrate these points in turn.

First, in standard VAR literature identification is achieved by implicitly assuming that the shocks and the related impulse response functions are "fundamental", i.e. that they are innovations with respect to the variables used in estimation. This assumption has weak economic motivations. An important argument against the fundamentalness assumption is that economic agents might use superior information with respect to the one used by the econometrician in the VAR. Small VARs are particularly subject to this criticism. The fundamentalness problem is well known in the literature (Lippi and Reichlin 1993, 1994) but has been largely ignored for the simple reason that there is no solution within the VAR approach. The only thing we can do with fundamentalness is to cross fingers and bet on it.

Second, the choice of the variables. Clearly, some variables must be included in the data set simply because they are the variables of interest for the problem at hand. However, further variables are often added with the motivation that they are related in some way to the variables of interest. Such variables, by enlarging the information set, may mitigate the problem of fundamentalness. However, the number and the nature of such variables is largely discretionary, and empirical results are not robust with respect to different choices.

Third, given the variables to include in the system, the identification scheme is often incredible, particularly for large systems. The number of equality restrictions to impose for a complete identification grows with the square of the number of variables. With 4 variables we have to impose 6 restrictions; with 5 variables we have to impose 10; with 6 variables, 15. As a consequence, when we have more than 3 or 4 variables, the economic theory can hardly provide enough restrictions to achieve identification, let alone testable implications. Even if we limit ourselves to triangular identification schemes, we have many different orderings, and the choice between them is not obvious at all. As a consequence, we often end up with restrictions which are difficult to interpret, and the relation between such restrictions and the labels attached to the shocks—“supply”, “demand”, etc.—are weak and questionable. The fact that adding variables renders identification more difficult is somewhat paradoxical, since intuition suggests that adding information should help identification rather than complicate it.

In this paper we explore the identification issue within a different class of models, including both the classical dynamic factor model or index model (Sargent and Sims, 1977, Geweke, 1977) and the generalization recently proposed by Forni et al. (2000).

So far, dynamic factor models have mainly been used as statistical tools, aimed at prediction or construction of economic indicators, rather than structural representation of economic relations. This is somewhat surprising, since such models are well suited for structural analysis.

The representation of macroeconomic variables emerging from dynamic factor models is very similar to that of Structural VARs. The basic difference is that we have two kinds of shocks instead of only one: the common or macroeconomic shocks, affecting all of the variables in the system, which play the same role as the shocks in structural VARs, and the idiosyncratic shocks, affecting exclusively, or almost exclusively, a specific variable. Within a macroeconomic context, such shocks must be interpreted essentially as measurement errors and short-run disturbances.

Macroeconomists wondering whether explicit modelling of measurement errors is really useful should remind that many macroeconomic variables such as the GDP are estimated, rather than merely observed or “measured”, so that “measurement error” is indeed a euphemism for “estimation error”. Moreover, there can be sources of variation which are not errors but nonetheless affect only a single variable or a small group of variables. As an example, think of short-run fluctuations of financial variables or exchange rates, which are not sufficiently long-lasting to pervade other portions of the economic activity.

The distinction between the true structural macroeconomic shocks, on one hand, and the noise generated by errors and disturbances, on the other hand, has the important consequence that the number of macroeconomic shocks is no longer constrained to be equal to the number of variables that we choose to analyze, a feature of SVARs which we find rather unpleasant. Within the factor model framework we can ask how many shocks are there in the macroeconomy, an interesting question which does not even make sense within the VAR framework.

But the crucial point is that typically the number of common shocks will be much smaller than the number of variables in the system, so that the relation between the amount of empirical data which can be handled by the model and the amount of information needed to achieve identification changes dramatically.

VARs cannot be very large, since with more than 10 or 15 variables the number of parameters to estimate is too large as compared with the number of time observation which are typical in empirical macroeconomics. By contrast, with factor models we can accommodate hundred of variables: virtually, we can manage all the existing macroeconomic information.

Of course, the choice of the data set is still important, but results are typically much more robust to the inclusion of an additional variable or a small group of variables in the data set. Moreover, we are enabled to study the effect of a shock upon many aggregate and disaggregate economic variables. Finally, as we shall see, the inclusion of a large number of time series enables us to estimate non-fundamental impulse response functions.

In factor models, unlike in structural VARs, when adding a variable the number of structural shocks does not change. As we shall see, a very small number of common shocks—just three—can provide a good representation of macroeconomic data. As a consequence, the number of restrictions which are needed to achieve identification is also small and the interpretation of the identifying restrictions and the shocks themselves is easier.

The paper is structured as follows. In Section 2, we present the moving-average factor model to which we refer in the sequel. In Section 3 we study the identification issue in the context of dynamic factor models and compare identification in factor and VAR models. We show that in both models, under the assumption of fundamentalness, the impulse-response functions and the structural shocks are identified up to static orthonormal rotations. Moreover, we discuss the severity of the fundamentalness restriction within the two theoretical frameworks and conclude that fundamentalness is much more acceptable within the factor model approach. In Section 4 we propose a method to estimate the impulse response functions and show consistency of the proposed estimator as both the time and the cross-sectional dimensions go to infinity. In Section 5 we provide an empirical illustration using a panel of 89 US quarterly macroeconomic series, specifically constructed to compare results with the three-variable model of King *et al.* (1991). We choose a three common shock specification and identify a permanent shock by imposing long-run neutrality of the other shocks on output. We find that (i) the three-dimensional sub-system of impulse response functions concerning the variables of King *et al.* (1991) is non-fundamental and therefore cannot be estimated with a VAR model; (ii) the impulse response functions are simple positive distributed lags and therefore do not have the implausible negative slope after two years found in King *et al.* (1991); (iii) the conclusion of King *et al.* (1991) that "US data are not consistent with the view that a single real permanent shock is the dominant source of business cycle fluctuations" is confirmed.

## 2 The Model

In this paper we refer to the following moving average dynamic factor model, which is a special case of the generalized dynamic factor model of Forni *et al.* (2000) and Forni and Lippi (2001). Such model, and the one used here, differs from the traditional dynamic factor model of Sargent and Sims (1977) and Geweke (1977), in that the number of cross-sectional variables is infinite and the idiosyncratic components are allowed to be mutually correlated to some extent, along the lines of Chamberlain (1983) and Chamberlain and Rothschild (1983). Similar models have been recently proposed by Stock and Watson (1998, 2002a, 2002b) and Bai and Ng (2002).

Denote by $\boldsymbol{x}_n^T = (x_{it})_{i=1,\dots,n;\,t=1,\dots,T}$ an $n \times T$ rectangular array of observations. We make two preliminary assumptions:

PA1. $\boldsymbol{X}_n^T$ is a finite realization of a real-valued stochastic process

$$\boldsymbol{X} = \{x_{it}, i \in \mathbb{N}, t \in \mathbb{Z}\,,\, x_{it} \in L_2(\Omega, \mathcal{F}, P)\}$$

indexed by $\mathbb{N}\times\mathbb{Z}$, where the $n$-dimensional vector processes $\{\boldsymbol{x}_{nt} = (x_{1t} \cdots x_{nt})',\ t \in \mathbb{Z}\}$, $n \in \mathbb{N}$ are stationary, with zero mean and finite second-order moments $\boldsymbol{\Gamma}_{nk} = \mathrm{E}[\boldsymbol{x}_{nt}\boldsymbol{x}'_{n,t-k}]$, $k \in \mathbb{N}$.

PA2. For all $n \in \mathbb{N}$, the process $\{\boldsymbol{x}_{nt}, t \in \mathbb{Z}\}$ admits a Wold representation $\boldsymbol{x}_{nt} = \sum_{k=0}^{\infty} C_k^n \boldsymbol{w}_{n,t-k}$, where the full-rank innovations $\boldsymbol{w}_{nt}$ have finite moments of order four, and the matrices $C_k^n = (C_{ij,k}^n)$ satisfy $\sum_{k=0}^{\infty} |C_{ij,k}^n| < \infty$ for all $n, i, j \in \mathbb{N}$.

We assume that the process $x_{it}$ is the sum of two unobservable components, the *common component* $\chi_{it}$ and the *idiosyncratic component* $\xi_{it}$. The common component is driven by a $q$-dimensional vector of *common shocks* $\boldsymbol{u}_t = (u_{1t}\ u_{2t}\ \cdots\ u_{qt})'$, which are loaded with possibly different coefficients and lags:

$$x_{it} = \chi_{it} + \xi_{it} = b_{i1}(L)u_{1t} + b_{i2}(L)u_{2t} + \cdots + b_{iq}(L)u_{qt} + \xi_{it}.$$

Note that $q$ is independent of $n$ (and small as compared to $n$ in empirical applications). More precisely:

FM0. Defining $\boldsymbol{\chi}_{nt} = (\chi_{1t}\ \ldots\ \chi_{nt})'$ and $\boldsymbol{\xi}_{nt} = (\xi_{1t}\ \ldots\ \xi_{nt})'$, and $B_n(L)$ as the matrix whose $(i,j)$ entry is $b_{ij}(L)$, we have

$$\begin{aligned} \boldsymbol{x}_{nt} &= \boldsymbol{\chi}_{nt} + \boldsymbol{\xi}_{nt} \\ &= B_n(L)\boldsymbol{u}_t + \boldsymbol{\xi}_{nt}, \end{aligned} \tag{2.1}$$

where $\boldsymbol{u}_t$ is an orthonormal white noise vector and $B_n(L) = B_0^n + B_1^n L + \ldots + B_s^n L^s$ is a $n \times q$ polynomial of order $s$ in the lag operator $L$. The matrices $B_j^n$ are nested as $n$ increases, and there is an $m$ such that $B_s^n \neq 0$ for $n > m$.

FM1. the process $\boldsymbol{u}_t$ is orthogonal to $\xi_{it},\ i = 1, \ldots, n,\ t \in \mathbb{Z}$.

Moreover, we make the following additional assumptions. Let $\Sigma_n^{\chi}(\theta)$, $\Sigma_n^{\xi}(\theta)$, $\theta \in [-\pi, \pi]$, be the spectral density matrices of $\boldsymbol{\chi}_{nt}$ and $\boldsymbol{\xi}_{nt}$, respectively, and $\lambda_{nk}^{\chi}$, $\lambda_{nk}^{\xi}$ the corresponding *dynamic eigenvalues*, namely, the mappings $\theta \mapsto \lambda_{nk}^{\chi}(\theta)$ and $\theta \mapsto \lambda_{nk}^{\xi}(\theta)$, where $\lambda_{nk}^{\chi}(\theta)$ and $\lambda_{nk}^{\xi}(\theta)$ stand for the $k$-th largest eigenvalues of $\Sigma_n^{\chi}(\theta)$ and $\Sigma_n^{\xi}(\theta)$, respectively. Finally, let $\Gamma_{nk}^{\chi}$ be the $k$-lag covariance matrix of $\boldsymbol{\chi}_{nt}$ and $\mu_{nj}^{\chi}$ the $j$-th eigenvalue of $\boldsymbol{\Gamma}_{n0}^{\chi}$.

FM2. For some $r$, $q \leq r \leq q(s+1)$, $\mu_{nr}^{\chi}(\theta) \to \infty$ as $n \to \infty$, $\theta$-a.e. in $[-\pi\ \pi]$;

FM3. There exists a real $\Lambda$ such that $\lambda_{n1}^{\xi}(\theta) \leq \Lambda$ for any $\theta \in [-\pi\ \pi]$ and any $n \in \mathbb{N}$;

FM4. $\lambda_{nk}^{\chi}(\theta) > \lambda_{n,k+1}^{\chi}(\theta)$ $\theta$-a.e. in $[-\pi\ \pi]$, $k = 1, \ldots, q$.

Assumptions FM2 and FM3 are needed to guarantee identification of the common and the idiosyncratic components (see Forni and Lippi, 2001). Note that condition FM3 on the asymptotic behavior of $\lambda_{nk}^{\xi}(\theta)$ includes the case in which the idiosyncratic components are mutually orthogonal with an upper bound for the variances. Mutual orthogonality is a standard, though highly unrealistic assumption in factor models; condition FM3 relaxes such assumption by allowing for a limited amount of cross-correlation among the idiosyncratic components. Assumptions PA2 and FM4 are technical and do not entail a severe loss of generality (see Forni *et al.*, 2002b for additional details).

It is easily seen that the moving average dynamic factor model above can be written in a "static" form, with common "factors" which are loaded only contemporaneously. Writing $\boldsymbol{f}_t$ for $(\boldsymbol{u}_t' \; \boldsymbol{u}_{t-1}' \; \dots \; \boldsymbol{u}_{t-s}')'$, we have

$$\boldsymbol{x}_{nt} = B_n(L)\boldsymbol{u}_t + \boldsymbol{\xi}_{nt} = A_n \boldsymbol{f}_t + \boldsymbol{\xi}_{nt} \tag{2.2}$$

with $r = q(s+1)$ "static" factors and $A_n = (\boldsymbol{a}_1' \; \cdots \; \boldsymbol{a}_n')' = (B_0^n \; B_1^n \; \cdots \; B_s^n)$.

In the sequel, we shall use the term *static factors* to denote the $r$ entries of $\boldsymbol{f}_t$ and the term *dynamic factors* to mean the $q$ entries of $\boldsymbol{u}_t$. Hence for instance $u_{1t}$ and $u_{1t-1}$ are two distinct static factors, but are different lags of the same dynamic factor.[1]

# 3 Identification

The results in this Sections hold both for the finite moving average factor model and the more general model proposed by Forni *et al.* (2000) and Forni and Lippi (2001); moreover, they can be trivially adapted to the traditional, finite $n$, dynamic factor model. What is relevant for the discussion is only that the common and the idiosyncratic components are uniquely characterized, whereas the particular set of assumptions ensuring identification is not essential. Given identification of the common components, we discuss identification of the common shocks $u_{ht}$, $h = 1, \dots, q$ and the impulse-response functions $b_{ih}(L)$, $h = 1, \dots, q$, $i \in \mathbb{N}$. A short preliminary description of the identification problem in Structural VAR models will be useful for comparison.

## 3.1 Structural VARs

Let $\boldsymbol{\chi}_t = (\chi_{1t} \; \cdots \; \chi_{qt})'$ be a zero-mean, covariance-stationary, $q$-dimensional, regular. Then $\boldsymbol{\chi}_t$ admits the moving average triangular representation

$$\boldsymbol{\chi}_t = B(L)\boldsymbol{u}_t \tag{3.3}$$

where

(VAR0) $B(L) = \sum_{k=0}^{\infty} B_k L^k$ is a $q \times q$ matrix of one-sided square-summable linear filters and $\boldsymbol{u}_t$ is as above a $q$-dimensional orthonormal white-noise vector process;

[1] The number of static factors is then the rank of the variance covariance matrix of the $\chi_{it}$'s, while the number of dynamic factors is the rank of the spectral density matrix of the $\chi_{it}$'s.

(VAR1) $\boldsymbol{u}_t$ is fundamental; i.e. $u_{ht}$, $h = 1, \ldots, q$, belong to the linear space spanned by the present and the past of $\chi_{ht}$, $h = 1, \ldots, q$;

(VAR2) $B(0) = B_0$ is lower triangular.

Such representation is called the *Cholesky-Sims triangular representation* and can be easily obtained from the Wold representation by using the Cholesky factorization of the covariance matrix of the Wold residuals. The triangular representation is unique, i.e. if $\boldsymbol{\chi}_t = C(L)\boldsymbol{v}_t$ with $C(L)$ and $\boldsymbol{v}_t$ fulfilling conditions VAR, then $C(L) = B(L)$ and $\boldsymbol{v}_t = \boldsymbol{u}_t$. Both existence and uniqueness of the triangular representation are immediate consequences of existence and uniqueness of the Wold representation.

**Remark 1**. Note that invertibility of $B(L)$ entails VAR1, since if $B(L)^{-1}$ exists, then $\boldsymbol{u}_t = B(L)^{-1}\boldsymbol{\chi}_t$. On the other hand, invertibility is not necessary for fundamentalness. For instance, if $q = 1$, $\chi_t = (1 - L)u_t$, $u_t$ is fundamental and VAR2 is fulfilled, but the representation is not invertible. Similarly, if $\boldsymbol{\chi}_t$ is the first difference of a vector of cointegrated variables, then $B(L)$, despite fulfilling VAR1, is not invertible since $\det B(1) = 0$.

We have infinitely many representations fulfilling conditions VAR0 and VAR1, but not VAR2. It can be shown that if

$$\boldsymbol{\chi}_t = C(L)\boldsymbol{v}_t, \tag{3.4}$$

where $\boldsymbol{v}_t$ is orthonormal and fundamental, then (3.4) is identified up to a *static rotation*, i.e. it is related to the triangular representation (3.3) by

$$\begin{aligned} C(L) &= B(L)H \\ \boldsymbol{v}_t &= H'\boldsymbol{u}_t, \end{aligned} \tag{3.5}$$

where $H$ is an orthonormal matrix, i.e. $HH' = I$. This is a well-known statement (a formal proof can be easily obtained along the lines of the Proposition in Section 3.3). Identification, within the standard VAR approach, consists in choosing $H$ such that economically motivated restrictions on the matrix $B(L)H$ are fulfilled. For instance, identification can be achieved by maximizing or minimizing an objective function involving $B(L)H$. But usually zero restrictions are imposed either on the impact effects $B(0)H$ or the long-run effects $B(1)H$ or both. In this case we have to impose $q(q-1)/2$ restrictions (since orthonormality entails $q(q+1)/2$ restrictions). Note that the triangular representations corresponding to different orderings of the variables in $\boldsymbol{\chi}_t$ can be obtained by imposing zero restrictions on the impact effects $B(0)H$.

### 3.2 Fundamentalness

The fundamentalness assumption VAR1 is crucial because, if we do not require it, the set of possible response functions increases enormously and identification become hopeless (Lippi and Reichlin 1993, 1994). If representation

$$\boldsymbol{\chi}_t = D(L)\boldsymbol{w}_t \tag{3.6}$$

fulfills VAR0 but not VAR1, it is identified up to *dynamic rotations*, i.e. it is related to the triangular representation by

$$\begin{aligned} D(L) &= B(L)H(L) \\ \boldsymbol{w}_t &= H'(F)\boldsymbol{u}_t, \end{aligned} \tag{3.7}$$

where $H(L)$ is a Blaschke matrix filter, i.e. a one-sided, square-summable linear filter such that $H(e^{-i\theta})H'(e^{i\theta}) = I$, $\theta$-a.e. in $[-\pi\ \pi]$.

**Example 1**. A simple example is

$$\chi_t = (1 + bL)u_t, \qquad |b| \leq 1.$$

Here $q = 1$, so that the condition $\text{var}(u_t) = 1$ is sufficient to identify the model. However, consider a representation of the form

$$\chi_t = d(L)w_t = (1 + bL)\frac{1 + hL}{h(1 + h^{-1}L)}w_t,$$

where

$$w_t = \frac{1 + hF}{h(1 + h^{-1}F)}u_t$$

and $|h| > 1$. Since $|h^{-1}| < 1$, $d(L)$ fulfills VAR3. Moreover, $w_t$ fulfills VAR0, since its spectral density is

$$\frac{1}{2\pi}\frac{1 + he^{i\theta}}{1 + h^{-1}e^{i\theta}}\frac{1 + he^{-i\theta}}{1 + h^{-1}e^{-i\theta}}\frac{1}{h^2} = \frac{1}{2\pi}$$

for any $\theta \in [-\pi\ \pi]$. However, $w_t$ is not contained in the information space spanned by the present and the past of $\chi_t$, because of the factor $(1 + hL)$, $|h| > 1$, in the numerator of $d(L)$. Hence $w_t$ is not fundamental, contrary to VAR1.

The basic argument in favor of fundamentalness is that it ensures that $\boldsymbol{u}_t$ is observable. It can be argued that structural macroeconomic shocks should be observable by economic agents, since otherwise they could not affect agents' behavior and produce effects on the macroeconomy. Since the macroeconomic variables in $\boldsymbol{\chi}_{t-k}$ are observable, the fundamentalness assumption entails that $\boldsymbol{u}_t$ is observable as well. However, also non-fundamental shocks can be observable, if agents have more information than that used by the econometrician, i.e. the present and the past of $\boldsymbol{\chi}_t$ alone (Quah 1990, Lippi and Reichlin 1993). With reference to the example above, if agents would observe $w_t$, $d(L)w_t$ would be the correct representation. We shall come back to this point below.

The fundamentalness assumption is necessary also in structural dynamic factor models. But we shall argue below that in the context of factor models fundamentalness is not particularly restrictive and can be convincingly justified.

## 3.3 Structural factor models

Now let us go back to the dynamic factor model (2.1). We neglect the idiosyncratic factors $\xi_{it}$ and concentrate on the common factors $\chi_{it}$, which are identified under Assumptions FM (see Section 2). We have in matrix notation

$$\boldsymbol{\chi}_{nt} = B_n(L)\boldsymbol{u}_t. \tag{3.8}$$

with the equality holding for any $n \in N$. We need the additional fundamentalness assumptions

(FM5) $\boldsymbol{u}_t$ is fundamental; i.e. $u_{ht}$, $h = 1, \ldots, q$ belong to the linear space spanned by the present and the past of $\chi_{it}$, $i = 1, \ldots, \infty$.

**Remark 2**. It should be observed that, if $r = q(s+1)$, the fundamentalness assumption FM5 is already implied by FM2. Clearly FM2 entails that, for $n$ sufficiently large, the rank of $\Gamma_n^\chi = A_n A_n'$ is equal to $r$, so that the rank of $A_n$ is also $r$ and an $r \times n$ matrix $R$ exists such that $RA_n = I_r$. Hence $R\boldsymbol{\chi}_{nt} = \boldsymbol{f}_t = (\boldsymbol{u}_t' \ \boldsymbol{u}_{t-1}' \ \ldots \ \boldsymbol{u}_{t-s}')'$, i.e. the factors can be generated as contemporaneous linear combinations of the $\chi_t$'s.

The following proposition holds:

**Proposition** If
$$\boldsymbol{\chi}_{nt} = C_n(L)\boldsymbol{v}_t \tag{3.9}$$
for any $n \in \mathbb{N}$ with the entries of $C_n(L)$ fulfilling FM0 and $\boldsymbol{v}_t$ fulfilling FM0 and FM5, then representation (3.9) is related to representation (3.8) by

$$\begin{aligned} C_n(L) &= B_n(L)H \\ \boldsymbol{v}_t &= H'\boldsymbol{u}_t, \end{aligned} \tag{3.10}$$

where $H$ is a $q \times q$ unitary matrix, i.e. $HH' = I_q$.

Proof. Projecting $\boldsymbol{v}_t$ entry by entry on the linear space $\mathcal{U}_t$ spanned by the present and the past of $u_{ht}$, $h = 1, \ldots, q$ we get

$$\boldsymbol{v}_t = \sum_{k=0}^{\infty} H_k \boldsymbol{u}_{t-k} + \boldsymbol{r}_t, \tag{3.11}$$

where $\boldsymbol{r}_t$ is orthogonal to $\boldsymbol{u}_{t-k}$, $k \geq 0$. Now consider that $\mathcal{U}_t$ and the space spanned by present and past of the $\chi_{it}$'s, call it $\mathcal{X}_t$, are identical, because the entries of $\boldsymbol{\chi}_{t-k}$, $k \leq 0$, belong to $\mathcal{U}_t$ by equation (3.9), while the entries of $\boldsymbol{u}_{t-k}$, $k \leq 0$, belong to $\mathcal{X}_t$ by Assumption FM5. The same is true for $\mathcal{X}_t$ and the space spanned by present and past of the $v_{ht}$'s, call it $\mathcal{V}_t$, so that $\mathcal{U}_t = \mathcal{V}_t$. Hence, by (3.11), $\boldsymbol{r}_t = 0$. Moreover, serial non-correlation of the $u_{ht}$'s imply that $\sum_{k=1}^{\infty} H_k \boldsymbol{u}_{t-k}$ must be the projection of $\boldsymbol{v}_t$ on $\mathcal{U}_{t-1}$, which is zero because $\mathcal{U}_{t-1} = \mathcal{V}_{t-1}$. It follows that $\boldsymbol{v}_t = H_0\boldsymbol{u}_t$. Orthonormality of $\boldsymbol{v}_t$ (Assumption FM0)implies that $H_0$ is unitary. QED

In the context of the dynamic factor model, the fundamentalness assumption is not particularly restrictive. To see this, consider the following sufficient condition.

(LI) For $n$ sufficiently large, there is a left-inverse for $B_n(L)$, i.e. a $n \times q$ one-sided filter $C_n(L)$ exists such that $C_n(L)'B_n(L) = I_q$.

Clearly if such a matrix exists, we have $\boldsymbol{u}_t = C_n(L)'\boldsymbol{\chi}_{nt}$ and FM5 holds. As we have already seen in Remark 2, the same sufficient condition holds for SVAR models. The basic difference is that here $n$ can be much larger than $q$. With $n$ large, invertibility becomes a mild condition. If a $q \times q$ invertible sub matrix of $B_n(L)$ exists, then of course $B_n(L)$ is invertible. If it does not, the left inverse could still exists. Consider the following example.

**Example 2**. Assume that $q = 1$ and that

$$\chi_{it} = b_i(1 - d_i L)u_t$$

with $d_i > 1$ for all $i$, so that there are no invertible sub matrices. Nonetheless, if $d_i \neq d_j$:

$$\frac{b_i(1 - d_i L)b_j d_j - b_j(1 - d_j L)b_i d_i}{(d_j - d_i)b_i b_j} = 1.$$

Therefore we can set $c_h(L) = 0$ for $h \neq i, j$;

$$c_i(L) = \frac{d_j}{(d_j - d_i)b_i}; \qquad c_j(L) = \frac{d_i}{(d_j - d_i)b_j}.$$

The only case in which we have non-fundamentalness is when $d_i = d$ for any $i$, so that

$$B_n(L) = B_n(0)(1 - dL)$$

with $|d| > 1$.

Example 2 clearly shows that fundamentalness of the whole system (FM5) does not imply fundamentalness (VAR1) of any $q \times q$ subsystem. Hence a non-fundamental impulse response subsystem, which cannot be estimated with a VAR, can in principle be identified and estimated within the factor model. As a matter of fact, in the empirical application below we estimate a non-fundamental impulse response function system, something which is impossible within the traditional approach.

Note also that the observability argument works differently for structural factor models and structural VARs. If agents look at all the macroeconomic information, they can observe (or, better, they can estimate consistently) the $\chi_{it}$'s, and therefore the $u_{ht}$'s. By contrast, if the econometrician takes just one macroeconomic variable, as in the example above, or a small subset of macroeconomic variables, as is usually done with SVARs, he does not have any guarantee that the structural impulse-response functions are fundamental with respect to this reduced information set.

**Remark 3**. It should be observed that we can still induce non-fundamentalness by means of dynamic Blaschke rotations. However, in the context of factor models, the impulse matrix $B_n(L)$ is $n \times q$, with $n$ large with respect to $q$. Hence, post multiplying by the $q \times q$ Blaschke matrix $H(L)$ like in equation (3.7) produces a plenty of restrictions on the way in which each cross-sectional unit reacts to the common shocks. Such kind of restrictions are hardly justified on theoretical grounds, and therefore should be considered of zero probability for any specific data set.

## 4 Estimation

If we were able to estimate the static factors $\boldsymbol{f}_t = (\boldsymbol{u}_t' \, \boldsymbol{u}_{t-1}' \, \dots \, \boldsymbol{u}_{t-s}')'$, we could estimate the impulse-response function simply by regressing the $x$'s on such estimated factors.

Unfortunately, we cannot estimate $\boldsymbol{f}_t$, since it is identified only up to pre-multiplication by a unitary matrix. The best we can do is to estimate the common-factor space, i.e. to estimate an $r$-dimensional, orthonormal vector whose entries span the same linear space as the entries of $\boldsymbol{f}_t$. Such vector can be written as $\boldsymbol{g}_t = G\boldsymbol{f}_t$, were $G$ is a non-singular matrix.

The static factor space can be consistently estimated by both the two-stage, generalized principal component estimator proposed by Forni *et al.* (2002b) and the principal component estimator proposed by Stock and Watson (2002a, 2002b). While Stock and Watson's principal component estimator is simpler, the two-stage estimator is more efficient in a number of cases (Forni *et al.*, 2002).[2]

To make things simple, the procedure proposed here is based on Stock and Watson's principal component estimator, i.e. we shall estimate the factor space by the first $r$ principal components of the panel $\boldsymbol{x}_{nt}$. Precisely, the estimated static factors will be

$$\hat{\boldsymbol{g}}_t = W_n^{T\prime}\boldsymbol{x}_{nt}, \tag{4.12}$$

where $W_n^T$ is the $n \times r$ matrix having on the columns the eigenvectors corresponding to the first $r$ largest eigenvalues of the sample variance-covariance matrix of $\boldsymbol{x}_{nt}$, say $\Gamma_{n0}^{xT}$. We do not normalize the factors to have unit variance. The estimated variance-covariance matrix of $\hat{\boldsymbol{g}}_t$ is the diagonal matrix having on the diagonal the eigenvalues $\Gamma_{n0}^{xT}$ in descending order, $\Lambda_n^T = W_n^{T\prime}\Gamma_{n0}^{xT}W_n^T$. The corresponding estimate of the common components is obtained by regressing $\boldsymbol{x}_{nt}$ on the estimated factors to get

$$\hat{\boldsymbol{\chi}}_{nt} = W_n^T W_n^{T\prime}\boldsymbol{x}_{nt}. \tag{4.13}$$

Having an estimate of $\boldsymbol{g}_t$, we have still to unveil the leading-lagging relations between its entries, in order to find out the underlying dynamic factors (or, better, a unitary transformation of such factors $\boldsymbol{v}_t = H\boldsymbol{u}_t$, with $HH' = I_q$). As shown below, this can be done in the moving average dynamic factor model by projecting $\boldsymbol{g}_t$ on its first lag. This approach is also followed in Giannone *et al.* (2002). The introduction of this dynamic dimension will produce not only an estimate of the impulse-response functions but also a new estimate of the $\chi$'s and a new estimate of the common (and idiosyncratic) variance-covariance matrices. This approach is also used

## 4.1 Population formulas

Going back to equation (2.2), it is seen that, by definition,

$$\boldsymbol{f}_t = F\boldsymbol{f}_{t-1} + \boldsymbol{e}_t,$$

where

$$F = \begin{pmatrix} \underset{(q \times sq)}{0} & \underset{(q \times q)}{0} \\ \underset{(sq \times sq)}{I} & \underset{(sq \times q)}{0} \end{pmatrix}$$

[2] Consistency of Stock and Watson's estimator for the model discussed here is proven in Forni *et al.* (2002b). For additional information on this topic see also Connor and Korajczyk (1988), Forni and Lippi (1997, 2001), Forni and Reichlin (1996, 1998, 2001), Forni *et al.* (2000, 2001, 2002a, 2002b), Stock and Watson (1998, 2002a, 2002b).

and

$$\boldsymbol{e}_t = \begin{pmatrix} \boldsymbol{u}_t \\ 0 \\ (sq \times 1) \end{pmatrix},$$

is orthogonal to $\boldsymbol{f}_{t-1}$. It follows that any non-singular transformation of the common factors $\boldsymbol{g}_t = G\boldsymbol{f}_t$ has the VAR(1) representation

$$\boldsymbol{g}_t = GFG^{-1}\boldsymbol{g}_{t-1} + \boldsymbol{\epsilon}_t = D\boldsymbol{g}_{t-1} + \boldsymbol{\epsilon}_t. \tag{4.14}$$

Note that

$$D = \Gamma_1^g \left(\Gamma_0^g\right)^{-1}, \tag{4.15}$$

where $\Gamma_h^g = \mathrm{E}(\boldsymbol{g}_t\boldsymbol{g}_{t-h}')$, and

$$\mathrm{var}(\boldsymbol{\epsilon}_t) = \Gamma_0^g - D\Gamma_0^g D'. \tag{4.16}$$

The residual $\boldsymbol{\epsilon}_t$ can be written as

$$\boldsymbol{\epsilon}_t = G\boldsymbol{e}_t = G_q\boldsymbol{u}_t = \left(G_qH'\right)H\boldsymbol{u}_t = KMH\boldsymbol{u}_t, \tag{4.17}$$

where

(i) $G_q$ is the $r \times q$ matrix formed by the first $q$ columns of $G$;

(ii) $M$ is the diagonal matrix having on the diagonal the square roots of the first $q$ largest eigenvalues of the variance-covariance matrix of $\boldsymbol{\epsilon}_t$, i.e. the matrix $G_qG_q' = \Gamma_0^g - D\Gamma_0^g D'$, in descending order.

(iii) $K$ is the $r \times q$ matrix whose columns are the eigenvectors corresponding to such eigenvalues.

(iv) $H$ is a $q \times q$ unitary matrix;

By inverting the VAR we get

$$\boldsymbol{g}_t = (I - DL)^{-1}KMH\boldsymbol{u}_t.$$

On the other hand, going back to equation (2.2) it is seen than

$$\boldsymbol{\chi}_{nt} = B_n(L)\boldsymbol{u}_t = A_n\boldsymbol{f}_t = A_nG^{-1}\boldsymbol{g}_t = Q_n\boldsymbol{g}_t, \tag{4.18}$$

where

$$Q_n = \mathrm{E}(\boldsymbol{\chi}_{nt}\boldsymbol{g}_t') = \mathrm{E}(\boldsymbol{x}_{nt}\boldsymbol{g}_t'). \tag{4.19}$$

Hence, we have

$$\begin{aligned} \boldsymbol{\chi}_{nt} &= B_n(L)\boldsymbol{u}_t \\ &= Q_n(I - DL)^{-1}KMH\boldsymbol{u}_t \\ &= Q_n(I + DL + D^2L^2 + \cdots)KMH\boldsymbol{u}_t \\ &= Q_n(I + DL + D^2L^2 + \cdots + D^sL^s)KMH\boldsymbol{u}_t, \end{aligned} \tag{4.20}$$

where the last equality can be obtained by observing that $\boldsymbol{\chi}_{nt}$ is orthogonal to $\boldsymbol{u}_{t-k}$ for $k > s$.

### 4.2 Estimators

By substituting $\hat{\boldsymbol{g}}_t = W_n^T \boldsymbol{x}_{nt}$ for $\boldsymbol{g}_t$, it is quite natural to estimate $Q_n$ by $\Gamma_{n0}^{xT} W_n^T$ (see equation (4.19)). Moreover, $\Gamma_0^g$, the variance-covariance matrix of $\boldsymbol{g}_t$, can be estimated by $W_n^{T\prime} \Gamma_{n0}^{xT} W_n^T = \Lambda_n^T$, and $\Gamma_1^g$ by $W_n^{T\prime} \Gamma_{n1}^{xT} W_n^T$, so that, basing on equation (4.15), we estimate $D$ by $D_n^T = W_n^{T\prime} \Gamma_{n1}^{xT} W_n^T \Lambda_n^{T-1}$. Finally, to estimate the eigenvectors and eigenvalues in $K$ and $M$ we estimate the variance-covariance matrix of $\boldsymbol{\epsilon}_t$ by $\Lambda_n^T - D_n^T \Lambda_n^T D_n^{T\prime}$ (see equation (4.16)).

Summing up, in analogy with (4.20) we propose to estimate the impulse-response functions by

$$B_n^T(L) = Q_n^T \left( I + D_n^T L + (D_n^T)^2 L^2 + \cdots + (D_n^T)^s L^s \right) K_n^T M_n^T H, \tag{4.21}$$

where

(i) $Q_n^T = \Gamma_{n0}^{xT} W_n^T$, where $\Gamma_{n0}^{xT}$ is the sample variance-covariance matrix of $\boldsymbol{x}_{nt}$ and $W_n^T$ the $n \times r$ matrix having on the columns the eigenvectors corresponding to the first $r$ largest eigenvalues of $\Gamma_{n0}^{xT}$;

(ii) $D_n^T = W_n^{T\prime} \Gamma_{n1}^{xT} W_n^T$, where $\Gamma_{n1}^{xT}$ is the sample covariance matrix of $\boldsymbol{x}_{nt}$ and $\boldsymbol{x}_{nt-1}$;

(iii) $M_n^T$ is the diagonal matrix having on the diagonal the square roots of the first $q$ largest eigenvalues of the the matrix $\Lambda_n^T - D_n^T \Lambda_n^T D_n^{T\prime}$, in descending order;

(iv) $K_n^T$ is the $r \times q$ matrix whose columns are the eigenvectors corresponding to such eigenvalues.

(v) $H$ is a unitary matrix to be fixed by the identifying restrictions.

Moreover, equations (4.17) and (4.14) motivate estimation of $\boldsymbol{u}_t$ by

$$\begin{aligned} \boldsymbol{u}_t^T &= H'(M_n^T)^{-1} K_n^{T\prime} \boldsymbol{\epsilon}_t^T \\ \boldsymbol{\epsilon}_t^T &= W_n^{T\prime} \boldsymbol{x}_{nt} - D_n^T W_n^{T\prime} \boldsymbol{x}_{nt-1}, \end{aligned} \tag{4.22}$$

where, to avoid confusion with $n$-dimensional vectors, we do not make explicit the dependence of $\boldsymbol{u}_t^T$ and $\boldsymbol{\epsilon}_t^T$ on $n$.

Note that the rank of the variance-covariance matrix of $\boldsymbol{\epsilon}_t^T$ will be $r$, not $q$. This is because, with a finite $n$, the principal components still have a (possibly very small) idiosyncratic term, which, when projecting on $\hat{\boldsymbol{g}}_{t-1}$, will enter the residuals. When taking the first $q$ (normalized) principal components of $\boldsymbol{\epsilon}_t^T$, we wash out such residual idiosyncratic elements. Hence imposing a dynamic structure on the estimated factors entails a new estimate of the factors themselves and the common components. Precisely, we have

$$\begin{aligned} \boldsymbol{g}_t^T &= D_n^T \boldsymbol{g}_{t-1}^T + K_n^T K_n^{T\prime} \boldsymbol{\epsilon}_t^T && (4.23) \\ &= \left( I + D_n^T L + (D_n^T)^2 L^2 + \cdots + (D_n^T)^s L^s \right) K_n^T M_n^T H \boldsymbol{u}_t^T && (4.24) \\ \boldsymbol{\chi}_{nt}^T &= B_n^T(L) \boldsymbol{u}_t^T && (4.25) \\ &= Q_n^T \left( I + D_n^T L + (D_n^T)^2 L^2 + \cdots + (D_n^T)^s L^s \right) K_n^T M_n^T H \boldsymbol{u}_t^T. \end{aligned}$$

The idiosyncratic components can be estimated by $\boldsymbol{\xi}_{nt}^T = \boldsymbol{x}_{nt} - \boldsymbol{\chi}_{nt}^T$.

Note also that, when $r$ is overestimated, the principal components in excess are mainly idiosyncratic, and the second 'washing' described above has a large effect. As a consequence, the 'corrected' estimate $\boldsymbol{\chi}_{nt}^T$ should be much less affected than $\hat{\boldsymbol{\chi}}_{nt}$ by overestimation of $r$, provided that $q$ is not itself overestimated.

Finally, corresponding to the above formula for $\boldsymbol{\chi}_{nt}^T$, the variance-covariance matrix of the common components can be estimated by

$$Q_n^T \left( C_n^T + D_n^T C_n^T D_n^{T\prime} + (D_n^T)^2 C_n^T (D_n^T)^{2\prime} + \cdots + (D_n^T)^s C_n^T (D_n^T)^{s\prime} \right) Q_n^{T\prime} \tag{4.26}$$

where $C_n^T = K_n^T (M_n^T)^2 K_n^{T\prime}$.

In order to render operative the above procedure we need to set values for $r$ and $q$. Unfortunately, there are no criteria in the literature to fix jointly $q$ and $r$. Bai and Ng (2002) propose some consistent criteria to determine $r$. As regards the number of dynamic factors, we can follow a decision rule like that proposed in Forni *et al.* (2000) i. e., we go on to add factors until the additional variance explained by the last factor is less than a pre-specified fraction, say 5% or 10%, of total variance.

### 4.3 Consistency

Consistency of the estimator (4.22) and (4.21) for $\boldsymbol{u}_t$ and the impulse-response functions respectively can be proved along the lines followed in Forni et al. (2002b), Section 5. Here we limit ourselves to provide an outline of the proof.

(a) Note firstly that the matrices $Q_n^T$, $D_n^T$, $K_n^T$, $M_n^T$, entering the definition of $B_n^T(L)$, see (4.21), all depend on the matrices $\Gamma_{nk}^{xT}$, their eigenvalues and eigenvectors. Under the assumption of no multiple eigenvalues (see Forni *et al.*, 2002b, for technical details), such matrices are therefore continuous functions of the coefficients of $\Gamma_{nk}^{xT}$.

(b) Given $n$, for $T \to \infty$ the estimators $\Gamma_{nk}^{xT}$ converge in probability to their population counterparts $\Gamma_{nk}^{x}$. Continuity implies that the matrices $Q_n^T$, $D_n^T$, $K_n^T$, $M_n^T$, and the shocks $\boldsymbol{u}_t^T$, converge in probability to $\check{Q}_n$, $\check{D}_n$, $\check{K}_n$, $\check{M}_n$, and the shocks $\check{\boldsymbol{u}}_t$, the latter being population matrices and shocks, where "population" here means that dependence on $T$ no longer holds, although these matrices and shocks still depend on $n$.

(c) It may be proved that as $n \to \infty$ the matrices $\check{Q}_n$, $\check{D}_n$, $\check{K}_n$, $\check{M}_n$, and the shocks $\check{\boldsymbol{u}}_t$ tend in variance to the population, with respect to both $T$ and $n$, matrices and shocks.

(d) Combining the asymptotics in $T$ with the asymptotics in $n$, it is then proved that there exist paths for $(T, n)$, with $T$ and $n$ both tending to infinity, such that along those paths the matrices $Q_n^T$, $D_n^T$, $K_n^T$, $M_n^T$, and the shocks $\boldsymbol{u}_t^T$ converge in probability to their population counterpart. Under additional assumptions Forni *et al.* (2002a) and Giannone *et al.* (2002) obtain convergence in probability to population values for $\min(n, T) \to \infty$. The results on the rate of convergence in

Forni *et al.* (2002a) can easily be adapted to the model analysed in the present paper.

(e) It should be pointed out that in the present paper estimation of the factors is based on the eigenvectors of the variance-covariance matrix of the $x$'s, not, as in Forni *et al.* (2000) or Forni *et al.* (2002b), on the eigenvectors of their spectral density matrix. Therefore the proof of consistency outlined above could alternatively be based, up to minor modifications of the assumptions, upon the methods used in Stock and Watson (1998) or in Bai and Ng (2002).

### 4.4 Standard errors and confidence bands

To obtain confidence bands and standard errors we propose the following bootstrap procedure.

First, compute $B_n^T(L)$ , $\boldsymbol{u}_t^T$ and $\boldsymbol{\chi}_t^T$ according to (4.21), (4.22) and (4.26), and $\boldsymbol{\xi}_{nt}^T = \boldsymbol{x}_{nt}^T - \boldsymbol{\chi}_{nt}^T$.

Second, for each one of the estimated idiosyncratic components, estimate the univariate autoregressive models

$$a_j(L)\chi_{jt}^T = \sigma_j \omega_{jt}, \qquad j = 1, \dots, n,$$

whose the order can be fixed by the Schwarz criterion, and take the estimated coefficients $a_j^T(L)$ and $\sigma_j^T$ and the unit variance residuals $\omega_{jt}^T$.

Third, generate new simulated series for the shocks, say $\boldsymbol{u}_t^*$ and $\omega_{jt}^*$, $j = 1, \dots, n$, either by drawing from the standard normal or by resampling from $\boldsymbol{u}_t^T$ and $\omega_{jt}^T$, $t = 1, \dots, T$. Use these new series to construct $\boldsymbol{\chi}_{nt}^* = B_n^T(L)\boldsymbol{u}_t^*$, $\xi_{jt}^* = a_j^T(L)^{-1}\sigma_j^T\omega_{jt}^*$, $j = 1, \dots, n$, and $\boldsymbol{x}_{nt}^* = \boldsymbol{\chi}_{nt}^* + \boldsymbol{\xi}_{nt}^*$.

Finally, compute new estimates of the impulse-response functions $B_n^*(L)$ starting from $\boldsymbol{x}_{nt}^*$.

By repeating the two last steps $N$ times we get a distribution of estimated values which can be used to obtain standard errors and confidence bands. Note that the estimates will in general be biased, since the estimation procedure involves implicitly the estimation of a VAR. An estimate of such bias is provided by the difference between the point estimate $B_n^T(L)$ and the average of the $N$ estimates $B_n^*(L)$.

## 5 Empirical application

We illustrate our proposed structural factor model by revisiting a seminal work in the structural VAR literature, i.e. King *et al.* (1991, KPSW from now on). To this end, we constructed a panel of macroeconomic series including the series used by KPSW, with the same sampling period. Just like KPSW, we identify a long-run shock by imposing long-run neutrality of all other shocks on per-capita output. The data are well described by three common shocks, so that the comparison with the three-variable exercise of KPSW is particularly appropriate. Having the same data, the same identification scheme and the same number of shocks, different results can only be due to the additional information coming from the other series in the panel.

## 5.1 The data

The data set was constructed by downloading mainly from the FRED II database of the Federal Reserve Bank of St. Louis and Datastream. The original data of KPSW have been downloaded from Mark Watson's home page. We collected 89 series, including data from NIPA tables, price indeces, productivity, industrial production indeces, interest rates, money, financial data, employment, labor costs, shipments, and survey data. A larger $n$ would be desirable, but we were constrained by both the scarcity of series starting from 1949 (like in KPSW) and the need of balancing data of different groups. In order to use Datastream series we were forced to start from 1950:1 instead of 1949:1, so that the sampling period is 1950:1 - 1988:4. Monthly data are taken in quarterly averages. All data have been transformed to reach stationarity according to the ADF(4) test at the 5% level. Finally, the data were taken in deviation from the mean as required by our formulas, and divided by the standard deviation to render results independent of the units of measurement. A complete description of each series and the related transformations is reported in the Appendix.

## 5.2 The choice of $r$ and the number of common shocks

As a first step we have to set $r$ and $q$. Let us begin by observing that in practice satisfying the constraint $r = (s+1)q$ is not convenient. An obvious reason is that, if $q > 1$, there could be shocks whose coefficients vanish after a lag smaller than $s$. More generally, there can be restrictions between the parameters enabling us to describe the impulse response functions more parsimoniously. As an example, consider the case $q = 1$ where there are only three kinds of shapes for the impulse-response functions of different cross-sectional units, say leading, lagging and coincident. In this case, $r = 3$ is sufficient to describe conveniently the data set, no matter the value of $s$.[3] As a matter of fact, assuming a finite $s$ is not really necessary. An example with a very small $r$ and infinite order response functions is the stylized equilibrium business cycle model studied in Giannone *et al.* (2003).

If we allow for $r < (s+1)q$, we can set $r$ and $q$ and let $s$ be whatever. Let us begin with $r$. We computed the six consistent criteria suggested by Bai and Ng (2002) with $r = 1, \dots, 30$. The criteria $IC_{p1}$ and $IC_{p3}$ do not work, since they do not reach a minimum for $r < 30$; $IC_{p2}$ has a minimum for $r = 12$. To compute $PC_{p1}$, $PC_{p2}$ and $PC_{p3}$ we estimated $\hat{\sigma}^2$ with $r = 15$ since with $r = 30$ none of the criteria reaches a minimum for $r < 30$. $PC_{p1}$ gives $r = 15$, $PC_{p2}$ gives $r = 14$ and $PC_{p3}$ gives $r = 20$. Hence we concentrated on the interval $12 \leq r \leq 20$.

For these values of $r$, and $1 \leq q \leq 6$, we used formula (4.26) to compute the variance explained by the common component for our main series of interest, i.e. real per capita output, and the whole system (Table 1). By adding the third shock the overall explained variance increases by 8-9 percentage points and the explained variance of per capita output by 4-8 per cent, as against the 4-5 per cent and 2-4 per cent respectively of the

[3] Notice however that if $r$ is strictly smaller than $(s+1)q$ we are no longer guaranteed that the static factors follow a VAR of order one. Hence looking at the serial correlation of the VAR residuals can be useful.

Table 1: **Percentage of variance explained by the common component**

| | $q = 1$ | $q = 2$ | $q = 3$ | $q = 4$ | $q = 5$ | $q = 6$ |
|---|---|---|---|---|---|---|
| *Average* | | | | | | |
| $r = 12$ | 0.19 | 0.31 | 0.39 | 0.44 | 0.47 | 0.51 |
| $r = 14$ | 0.19 | 0.30 | 0.38 | 0.43 | 0.46 | 0.51 |
| $r = 16$ | 0.19 | 0.31 | 0.39 | 0.43 | 0.46 | 0.51 |
| $r = 18$ | 0.18 | 0.30 | 0.38 | 0.43 | 0.47 | 0.51 |
| $r = 20$ | 0.18 | 0.30 | 0.39 | 0.43 | 0.47 | 0.51 |
| *Per capita output* | | | | | | |
| $r = 12$ | 0.31 | 0.47 | 0.53 | 0.55 | 0.56 | 0.59 |
| $r = 14$ | 0.33 | 0.48 | 0.52 | 0.55 | 0.56 | 0.58 |
| $r = 16$ | 0.33 | 0.48 | 0.53 | 0.55 | 0.57 | 0.58 |
| $r = 18$ | 0.31 | 0.46 | 0.53 | 0.57 | 0.59 | 0.61 |
| $r = 20$ | 0.31 | 0.46 | 0.54 | 0.56 | 0.59 | 0.60 |

fourth shock. As explained above, for the sake of comparison we start with a strong preference in favor of $q = 3$. The numbers above are not at odds with this choice. It is worth noting that Giannone *et al.* (2002) also set $q = 3$ with a larger data set referring to a more recent period.

Regarding the choice of $r$, both the criteria above and the explained variances of Table 1 do not provide a definite answer. However, as we shall see, results are quite robust with respect to variation of $r$. Below we report results for different values of $r$, with more detailed statistics for $r = 15$.

### 5.3 Fundamentalness

Now let us compute the roots of the determinant of the impulse-response function system formed by the three variables of KPSW, i.e. per capita consumption, per capita income and per capita investment.[4] Figure 1 plots the moduli of the two smallest roots of the above determinant as a function of $r$, for $r$ varying over the range 3-30. Note that for $r = 3$ all roots must be larger than one in modulus, since they stem from a three-variate VAR. This is in fact the case for $r = 3$ and $r = 4$, but for $r \geq 5$ the smallest root is declining and lies always within the unit circle. For $r \geq 22$ the second smallest root becomes smaller than one in modulus.

Figure 2 reports the distribution of the modulus of the smallest root for $r = 15$ across 1000 bootstrapping replications. The mean value is 0.71, indicating a non-negligible upward bias, since our point estimate for $r = 15$ is 0.54. We shall come back to the estimation bias below. Here we limit ourselves to observe that if the smallest root is overestimated on average, the true value could be even smaller than 0.54. Without any bias correction, the probability of an estimated value larger than one in modulus is less than 22%.

[4] Note that these roots (and therefore fundamentalness) are independent of the identification rule adopted and the rotation matrix $H$.

Figure 1: **The moduli of the first and the second smallest roots as functions of $r$**

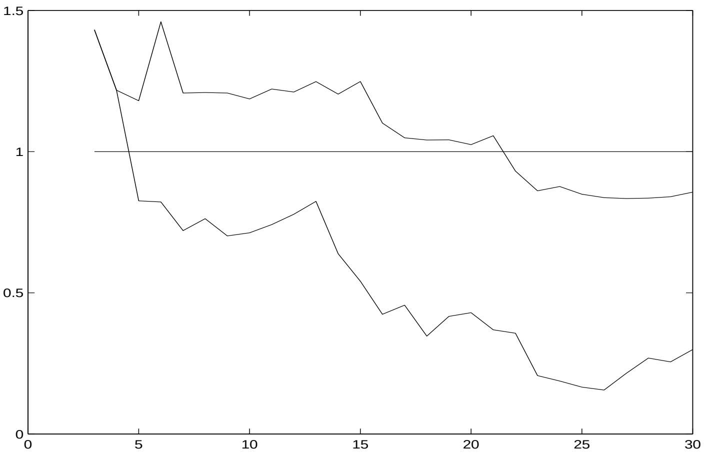
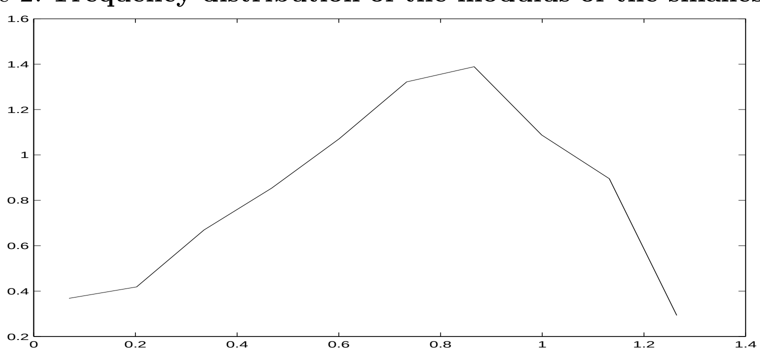



Figure 2: **Frequency distribution of the modulus of the smallest root**



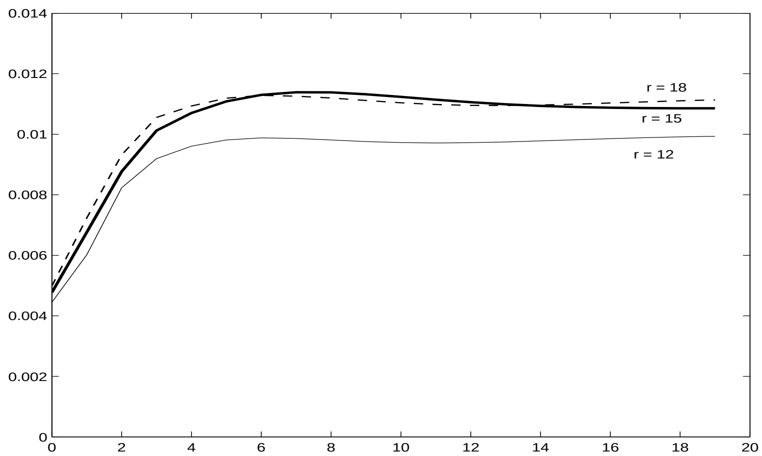

We conclude that the true impulse-response functions for the three variable system of KPSW are probably non-fundamental and therefore cannot be estimated with traditional VAR techniques.

## 5.4 Impulse-response functions and variance decomposition

Coming to the impulse-response functions, as anticipated above we impose long-run neutrality of two shocks on per-capita output, like in KPSW. This is sufficient to reach a partial identification, i.e. to identify the long-run shock and its response functions on the three variables.

Figure 3 shows the response functions of per capita output for $r = 12, 15, 18$. The general shape does not change that much with $r$. The productivity shock has positive effects declining with time on the output level. The response function reach its maximum value after 6-8 quarters with only negligible effects after two years. This shape is very different from the one in KPSW, where there is a sharp decline during the second and the third year which drives the overall effect back to the impact value. In our opinion such negative effects are not easily justified on theoretical grounds and classical distributed lags like the ones of Figure 3 are more convincing.

Figure 3: **The impulse response function of the long-run shock on output for** $r = 12, 15, 18$




In Figure 4 we concentrate on the case $r = 15$. We report the response functions with 90% confidence bands for output, consumption and investment respectively. Confidence bands are obtained with the nonparametric procedure explained above (with 1000 replications). The shapes are similar for the three variables, with a positive impact effect followed by important, though declining, positive lagged effects. Again, we do not have the large negative lagged effects found by KPSW particularly for investment.

Note that confidence bands are not centered around the point estimate, especially for consumption, suggesting the existence of a non-negligible bias. This is not surprising, since formula (4.21) implicitly involves estimation of a VAR, where in addition the variable involved (the static factors) contain errors (a residual idiosyncratic term). Figure 5 shows the point estimate along with the mean of the bootstrap distribution for the output. Such a large bias is probably due to the small cross-sectional dimension. We have evidence of a much smaller bias for the larger data set of Giannone *et al.* (2002). We do not make any attempt here to correct for the bias, but a procedure like the one suggested in Kilian (1998) could be appropriate.

Coming to variance decomposition, the percentage of the total variance of the common component attributable to the permanent shock is only 36.4% for output, 21.3% for investment and 38.8% for consumption.

Table 2 reports the fraction of the forecast-error variance attributed to the permanent shock for output, consumption and investment at different horizons. For ease of comparison we report the corresponding numbers obtained with the (restricted) VAR model and reported in Table 4 of KPSW.

At horizon 1, our estimates are smaller. The difference is important for consumption: only 0.30 according to the factor model as against 0.88 according to the KPSW model. But at horizons larger than or equal to 8 our estimates are greater and the difference is very large for investment. The basic conclusions of KPSW, however, are confirmed: "US data are not consistent with the view that a single real permanent shock is the dominant source of business-cycle fluctuations" (KPSW, p.838).

Figure 4: **The impulse response function of the long-run shock on output, consumption and investment for $r = 15$**

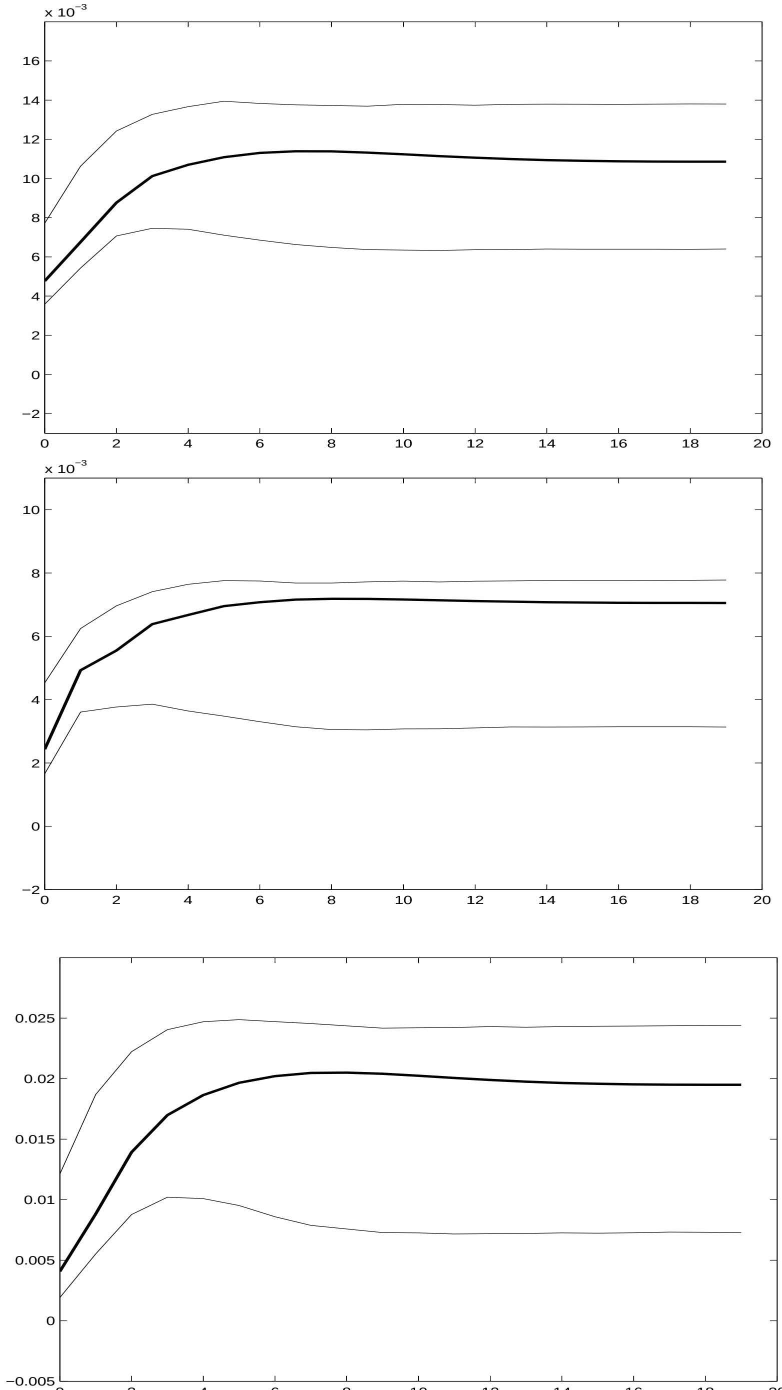
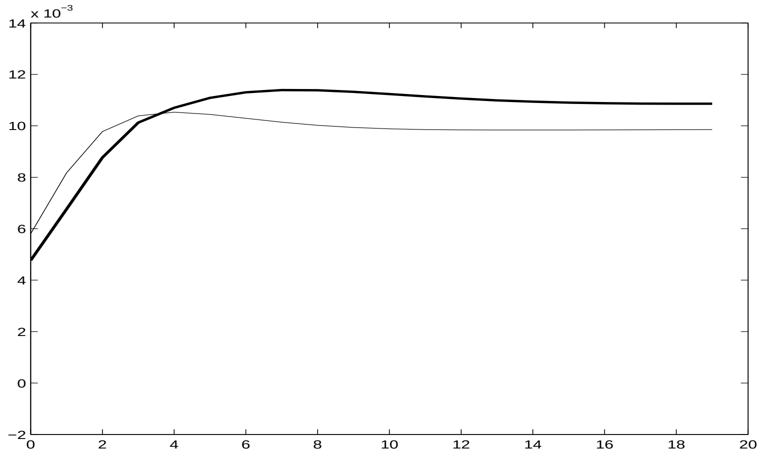


Figure 5: **Estimation bias**




# 6 Conclusions

In this paper we have argued that dynamic factor models are suitable for structural macroeconomic modeling and in some respects are preferable to structural VARs.

We have discussed identification within a dynamic factor model and have compared identification conditions within the two classes of models. In particular, we have argued that the usual fundamentalness assumption, which is necessary in both frameworks, is much less restrictive within the factor model context and can be better justified on economic grounds.

Having established sufficient conditions for identification, we have suggested a procedure in order to estimate the impulse response functions, based on Stock and Watson's principal component estimation of the (static) factor space. Moreover, we have shown consistency of such a procedure and have suggested a bootstrapping procedure for confidence bands and inference purposes.

In the empirical application, we have revisited the seminal paper by King *et al.* (1991, KPSW). We have designed a data set including the data of KPSW, with the same sample period. For the sake of comparison, we have chosen a three-shock specification and have imposed the same identification scheme as in KPSW.

First, we have found that the smallest root of the determinant of the impulse-response function system formed by the three variables of KPSW is non-fundamental and therefore cannot be obtained by estimating a VAR. This result is robust with respect to the choice of the static rank $r$.

Second, the shapes of the impulse-response functions of the long-run shock on output, investment and consumption are cumulated sums of simple positive distributed lags, and do not present the strange negative slope after the fourth quarter found by KPSW.

Third, the fraction of variance explained by the permanent shock is smaller in the very short run, particularly for consumption and larger after two years, particularly for investment. However, the basic conclusions of KPSW concerning the role of the

Table 2: **Fraction of the forecast-error variance due to the long-run shock**

| | Dynamic factor model | | | KPSW vector ECM | | |
|---|---|---|---|---|---|---|
| Horizon | Output | Cons. | Inv. | Output | Cons. | Inv. |
| 1 | 0.37 | 0.30 | 0.07 | 0.45 | 0.88 | 0.12 |
| | (0.18) | (0.21) | (0.19) | (0.28) | (0.21) | (0.18) |
| 4 | 0.57 | 0.77 | 0.42 | 0.58 | 0.89 | 0.31 |
| | (0.12) | (0.12) | (0.19) | (0.27) | (0.19) | (0.23) |
| 8 | 0.78 | 0.87 | 0.72 | 0.68 | 0.83 | 0.40 |
| | (0.07) | (0.11) | (0.16) | (0.22) | (0.18) | (0.18) |
| 12 | 0.86 | 0.90 | 0.80 | 0.73 | 0.83 | 0.43 |
| | (0.05) | (0.11) | (0.16) | (0.19) | (0.18) | (0.17) |
| 16 | 0.89 | 0.91 | 0.83 | 0.77 | 0.85 | 0.44 |
| | (0.04) | (0.11) | (0.16) | (0.17) | (0.16) | (0.16) |
| 20 | 0.91 | 0.92 | 0.86 | 0.79 | 0.87 | 0.46 |
| | (0.03) | (0.11) | (0.16) | (0.16) | (0.15) | (0.16) |

permanent shock in explaining the short-run volatility of output remain unchanged.

# Appendix: Data description and data treatment

| Database | Original Source | Variable Description | ID Code in the Database | Units | Orig. Freq. | Seas. Adj. | Treatment |
|---|---|---|---|---|---|---|---|
| 1 MW | Citibase | Per Capita Real Consumption Expenditure | | | | | DLOG |
| 2 MW | Citibase | Per Capita Gross Private Domestic Fixed Investment | | | | | DLOG |
| 3 MW | Citibase | Per Capita Private Gross National product | | | | | DLOG |
| 4 MW | Citibase | Per Capita Real M2 (M2 divided by P) | | | | | DLOG |
| 5 MW | Citibase | 3-Month Treasury Bill Rate | | | | | D |
| 6 MW | Citibase | Implicit Price Deflator for Private GNP | | | | | DDLOG |
| 7 Fred II | BEA | Real Gross Domestic Product, 1 Decimal | GDPC1 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 8 Fred II | BEA | Real Final Sales of Domestic Product, 1 Decimal | FINSLC1 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 9 Fred II | BEA | Real Gross Private Domestic Investment, 1 Decimal | GPDIC1 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 10 Fred II | BEA | Real State & Local Cons. Expend. & Gross Inv., 1 Dec. | SLCEC1 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 11 Fred II | BEA | Real Private Residential Fixed Investment, 1 Dec. | PRFIC1 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 12 Fred II | BEA | Real Private Nonresidential Fixed Investment, 1 Dec. | PNFIC1 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 13 Fred II | BEA | Real Nonresidential Inv.: Equipment & Software, 1 Dec. | NRIPDC1 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 14 Fred II | BEA | Real Imports of Goods & Services, 1 Decimal | IMPGSC1 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 15 Fred II | BEA | Real Federal Cons. Expend. & Gross Investment, 1 Dec. | FGCEC1 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 16 Fred II | BEA | Real Government Cons. Expend. & Gross Inv., 1 Dec. | GCEC1 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 17 Fred II | BEA | Real Fixed Private Domestic Investment, 1 Decimal | FPIC1 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 18 Fred II | BEA | Real Exports of Goods & Services, 1 Decimal | EXPGSC1 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 19 Fred II | BEA | Real Change in Private Inventories, 1 Decimal | CBIC1 | Bil. of Ch. 1996 $ | Q | YES | NONE |
| 20 Fred II | BEA | Real Personal Cons. Expenditures: Nondurable Goods | PCNDGC96 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 21 Fred II | BEA | Real State & Local Government: Gross Investment | SLINVC96 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 22 Fred II | BEA | Real Personal Consumption Expenditures: Services | PCESVC96 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 23 Fred II | BEA | Real Personal Cons. Expenditures: Durable Goods | PCDGCC96 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 24 Fred II | BEA | Real Personal Consumption Expenditures | PCECC96 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 25 Fred II | BEA | Real National Defense Gross Investment | DGIC96 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 26 Fred II | BEA | Real Federal Nondefense Gross Investment | NDGIC96 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 27 Fred II | BEA | Real Disposable Personal Income | DPIC96 | Bil. of Ch. 1996 $ | Q | YES | DLOG |
| 28 Fred II | BEA | Personal Cons. Expenditures: Chain-type Price Index | PCECTPI | Index 1996 = 100 | Q | YES | DDLOG |
| 29 Fred II | BEA | Gross Domestic Product: Chain-type Price Index | GDPCTPI | Index 1996 = 100 | Q | YES | DDLOG |
| 30 Fred II | BEA | Gross Domestic Product: Implicit Price Deflator | GDPDEF | Index 1996 = 100 | Q | YES | DDLOG |
| 31 Fred II | BEA | Gross National Product: Implicit Price Deflator | GNPDEF | Index 1996 = 100 | Q | YES | DDLOG |
| 32 Fred II | BEA | Gross National Product: Chain-type Price Index | GNPCTPI | Index 1996 = 100 | Q | YES | DDLOG |
| 33 Fred II | BLS | Nonfarm Business Sector: Unit Labor Cost | ULCNFB | Index 1996 = 100 | Q | YES | DLOG |
| 34 Fred II | BLS | Nonfarm Business Sector: Real Compensation Per Hour | COMPRNFB | Index 1992 = 100 | Q | YES | DLOG |
| 35 Fred II | BLS | Nonfarm Bus. Sector: Output Per Hour of All Persons | OPHNFB | Index 1992 = 100 | Q | YES | DLOG |
| 36 Fred II | BLS | Nonfarm Business Sector: Compensation Per Hour | COMPNFB | Index 1992 = 100 | Q | YES | DLOG |
| 37 Fred II | BLS | Manufacturing Sector: Unit Labor Cost | ULCMFG | Index 1992 = 100 | Q | YES | DLOG |
| 38 Fred II | BLS | Manufacturing Sector: Output Per Hour of All Persons | OPHMFG | Index 1992 = 100 | Q | YES | DLOG |
| 39 Fred II | BLS | Business Sector: Output Per Hour of All Persons | OPHPBS | Index 1992 = 100 | Q | YES | DLOG |
| 40 Fred II | BLS | Business Sector: Compensation Per Hour | HCOMPBS | Index 1992 = 100 | Q | YES | DLOG |
| 41 Fred II | St. | Louis St. Louis Adjusted Reserves | ADJRESSL | Bil. of $ | M | YES | DLOG |
| 42 Fred II | St. Louis | St. Louis Adjusted Monetary Base | AMBSL | Bil. of $ | M | YES | DLOG |
| 43 Fred II | Moody's | Moody's Seasoned Aaa Corporate Bond Yield | AAA | % | M | NO | D |
| 44 Fred II | Moody's | Moody's Seasoned Baa Corporate Bond Yield | BAA | % | M | NO | D |
| 45 Fred II | FR | Bank Prime Loan Rate | MPRIME | % | M | NO | D |
| 46 Fred II | FR | 3-Month Treasury Bill: Secondary Market Rate | TB3MS | % | M | NO | D |
| 47 Fred II | FR | Currency in Circulation | CURRCIR | Bil. of $ | M | NO | DD4LOG |
| 48 Fred II | FR | Currency Component of M1 | CURRSL | Bil. of $ | M | YES | DDLOG |
| 49 Fred II | BLS | CPI for All Urban Consumers: All Items Less Food | CPIULFSL | Ind. 1982-84 = 100 | M | YES | DDLOG |
| 50 Fred II | BLS | Consumer Price Index for All Urban Consumers: Food | CPIUFDSL | Ind. 1982-84 = 100 | M | YES | DDLOG |
| 51 Fred II | BLS | CPI For All Urban Consumers: All Items | CPIAUCSL | Ind. 1982-84 = 100 | M | YES | DDLOG |
| 52 Fred II | BLS | CPI: Intermediate Materials: Supplies & Components | PPIITM | Index 1982 = 100 | M | YES | DDLOG |
| 53 Fred II | BLS | Producer Price Index: Industrial Commodities | PPIIDC | Index 1982 = 100 | M | NO | DDLOG |
| 54 Fred II | BLS | PPI: Fuels & Related Products & Power | PPIENG | Index 1982 = 100 | M | NO | DDLOG |
| 55 Fred II | BLS | PPI Finished Goods: Capital Equipment | PPICPE | Index 1982 = 100 | M | YES | DDLOG |
| 56 Fred II | BLS | Producer Price Index: Finished Goods | PPIFGS | Index 1982 = 100 | M | YES | DDLOG |
| 57 Fred II | BLS | Producer Price Index: Finished Consumer Goods | PPIFCG | Index 1982 = 100 | M | YES | DDLOG |
| 58 Fred II | BLS | Producer Price Index: Finished Consumer Foods | PPIFCF | Index 1982 = 100 | M | YES | DDLOG |
| 59 Fred II | BLS | PPI: Crude Materials for Further Processing | PPICRM | Index 1982 = 100 | M | YES | DDLOG |
| 60 Fred II | BLS | Producer Price Index: All Commodities | PPIACO | Index 1982 = 100 | M | NO | DLOG |
| 61 Fred II | FR | Commercial and Industrial Loans at All Comm. Banks | BUSLOANS | Bil. of $ | M | YES | DLOG |
| 62 Fred II | FR | Total Loans and Leases at Commercial Banks | LOANS | Bil. of $ | M | YES | DLOG |
| 63 Fred II | FR | Total Loans and Investments at All Commercial Banks | LOANINV | Bil. of $ | M | YES | DLOG |
| 64 Fred II | FR | Total Consumer Credit Outstanding | TOTALSL | Bil. of $ | M | YES | DLOG |
| 65 Fred II | FR | Real Estate Loans at All Commercial Banks | REALLN | Bil. of $ | M | YES | DLOG |
| 66 Fred II | FR | Other Securities at All Commercial Banks | OTHSEC | Bil. of $ | M | YES | DLOG |
| 67 Fred II | FR | Consumer (Individual) Loans at All Comm. Banks | CONSUMER | Bil. of $ | M | YES | DLOG |
| 68 Fred II | BLS | All Employees: Construction | USCONS | Thous. | M | YES | DLOG |
| 69 Fred II | BLS | Total Nonfarm Payrolls: All Employees | PAYEMS | Thous. | M | YES | DLOG |
| 70 Fred II | BLS | Employees on Nonfarm Payrolls: Manufacturing | MANEMP | Thous. | M | YES | DLOG |
| 71 Fred II | BLS | Unemployed: 16 Years & Over | UNEMPLOY | Thous. | M | YES | DLOG |
| 72 Fred II | BLS | Civilian Unemployment Rate | UNRATE | % | M | YES | DLOG |
| 73 Fred II | BLS | Civilian Participation Rate | CIVPART | % | M | YES | DLOG |
| 74 Fred II | BLS | Civilian Labor Force | CLF16OV | Thous. | M | YES | DLOG |
| 75 Fred II | BLS | Civilian Employment: Sixteen Years & Over | CE16OV | Thous. | M | YES | DLOG |
| 76 Fred II | BLS | Civilian Employment-Population Ratio | EMRATIO | % | M | YES | DLOG |

| Database | Original Source | Variable Description | ID Code in the Database | Units | Orig. Freq. | Seas. Adj. | Treatment |
|---|---|---|---|---|---|---|---|
| 77 EconStats | FR | Industrial Production: total | Index | | M | YES | DLOG |
| 78 EconStats | FR | Industrial Production: Manufacturing (SIC-based) | Index | | M | YES | DLOG |
| 79 Datastream | ISM | ISM Manufacturers Survey: Supplier Delivery Index | USNAPMDL | Index | M | YES | NONE |
| 80 Datastream | ISM | Chicago Purchasing Manager Business Barometer | USPMCUBB | % | M | NO | NONE |
| 81 Datastream | ISM | ISM Manufacturers Survey: New Orders Index | USNAPMNO | Index | M | YES | NONE |
| 82 Datastream | ISM | ISM Manufacturers Survey: Employment Index | USNAPMIV | Index | M | YES | NONE |
| 83 Datastream | ISM | ISM Manufacturers Survey: Production Index | USNAPMEM | Index | M | YES | NONE |
| 84 Datastream | ISM | ISM Purchasing Managers Index (MFG Survey) | USNAPMPR | Index | M | YES | NONE |
| 85 Datastream | BC | Manufacturing Shipments - Total | USMNSHIPB | Bil. of $ | M | YES | DLOG |
| 86 Datastream | BC | Shipments of Durable Goods | USSHDURGB | Bil. of $ | M | YES | DLOG |
| 87 Datastream | BC | Shipments of Non-Durable Goods | USSHNONDB | Bil. of $ | M | YES | DLOG |
| 88 Datastream | S&P | Standard & Poor's 500 (monthly average) | US500STK | Index | M | NO | DLOG |
| 89 Datastream | FT | Dow Jones Industrial Share Price Index | USSHRPRCF | Index | M | NO | DLOG |

Abbreviations:
MW: Mark Watson's home page (http://www.wws.princeton.edu/ mwatson/publi.html)
Fred II: Fred II database of the Federal Reserve Bank of St. Louis
BEA: Bureau of Economic Analysis
BLS: Bureau of Labor Statistics
FR: Federal Reserve Board
St Louis: Federal Reserve Bank of St. Louis
ISM: Institute for Supply Management
BC: Bureau of Census
S&P: Standard & Poors'
FT: Financial Times
Q: Quarterly
M: Monthly (we take quarterly averages)

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